St. Xavier's College – Autonomous
Mumbai

Syllabus
For 3rd Semester Courses in MATHEMATICS

(2021 - 2022)

Contents:

S.Y.B.Sc. – Mathematics **Course Code: S.MAT.3.01**

Title: CALCULUS – III

Learning Objectives: To learn about (i) Riemann Integration **(ii) Different coordinate systems, Sketching in Improper integrals, β and ┌ functionsR^2 and R^3, iii) double integrals and its applications**

Number of lectures : 45

Unit I: Riemann Integration **(15 Lectures)**

Approximation of area,Upper / Lower Riemann sums and properties, Upper / Lower integrals, Definition of Riemann integral on a closed and bounded interval, Criterion for Riemann integrability, For a < c < b, f ∈ R[a, b] if and only if f ∈ R[a, c] and f ∈ R[c, b] with $\int_a^b f = \int_a^c f + \int_c^b f$. <u>Properties</u>: f, g ∈ R[a, b] then λ f ∈ R[a, b] and f+g ∈ R[a, b] with $\int_a^b \lambda f = \lambda \int_a^b f$ and $\int_a^b f + g = \int_a^b f + \int_a^b g$ and; f ∈ R[a, b] $|f| \in$ R[a, b]; $\left|\int_a^b f\right| \leq \int_a^b |f|$; $f \geq 0$ $\int_a f \geq 0$; f ∈ C[a, b] f ∈ R[a, b]; if f is bounded with finite number of discontinuities then f ∈ R[a, b] ; generalize this if f is monotone then f ∈ R[a, b].

Unit II: Indefinite and improper integrals **(15 Lectures)**

Continuity of $F(x) = \int_a^x f(t)dt$ where f ∈ R[a, b], Fundamental theorem of calculus,Mean value theorem, Integration by parts, Leibnitz rule, Improper integrals – type 1 and type 2, Absolute convergence of improper integrals, Comparison tests, Abel's and Dirichlet's tests (without proof); β and ┌ functions with their properties, relationship between β and ┌ functions.

Unit III: Applications **(15 Lectures)**

Topics from analytic geometry – sketching of regions in R^2 and R^3, graph of a function, level sets, Cylinders and Quadric surfaces, Cartesian coordinates, Polar coordinates, Spherical coordinates, Cylindrical coordinates and conversions from one coordinate system to another. (a) Double integrals: Definition of double integrals over rectangles, properties, double integrals

over a bounded region.
(b) Fubini`s theorem (without proof) – iterated integrals, double integrals as volume.
(c) Application of double integrals: average value, area, moment, center of mass.
(d) Double integral in polar form.
(Reference for Unit III: Sections 5.1, 5.2, 5.3 and 5.5 from Marsden-Tromba-Weinstein).

References:

1. Goldberg, Richard R.: Methods of real analysis. (2nd ed.) New York. John Wiley & Sons, Inc., 1976. 0-471-31065-4--(515.8GOL)

2. Goldberg, Richard R.: Methods of real analysis. (1st ed. Indian Reprint) New Delhi. Oxford & IBH Publishing Co., 1964(1975).--(515.8GOL)

3. Kumar, Ajit & Kumaresan, S.: A basic course in real analysis. (Indian reprint)

4. Boca Raton. CRC Press, 2015. 978-1-4822-1637-0--(515.8Kum/Kum)

5. Apostol, Tom M.: Calculus, Vol.-II. [Multi-variable calculus and linear algebra, with applications to differential equations and probability] (2nd ed.) New York. John Wiley & Sons, Inc., 1969. 0-471-00008-6--(515.14APO)

6. Stewart, James: Multivariable calculus:concepts & contexts. Pacific Grove.Brooks/Cole Publishing Company, 1998. 0-534-35509-9--(515STE)

7. Marsden, Jerrold E.;Tromba, Anthony J. & Weinstein, Alan: Basic multivariablecalculus.

   (Indian reprint) New Delhi. Springer (India) Private Limited, 1993(2004). 81-8128-186-1--(515.84MAR)

8. Robert G. & Sherbert, Donald R.: Introduction to real analysis. (3rd ed.) New Delhi. Wiley India (P) Ltd, 2005(2007). 81-265-1109-5--(515.8Bar/She)

Suggested Tutorials:

1. Calculation of upper sum, lower sum and Riemann integral.
2. Problems on properties of Riemann integral.
3. Problems on fundamental theorem of calculus, mean value theorems, integration by parts, Leibnitz rule.

4. Convergence of improper integrals, applications of comparison tests, Abel's and Dirichlet's tests.
5. Sketching of regions in R^2 and R^3, graph of a function, level sets, Cylinders and Quadric surfaces, conversions from one coordinate system to another.
6. Double integrals, iterated integrals, applications to compute average value, area, moment, center of mass.

****

S.Y.B.Sc. – Mathematics **Course Code: S.MAT.3.02**

Title: ALGEBRA - III

Learning Objectives: (i) To understand linear maps and isomorphism between vector spaces

(ii) To study determinant function via permutations and its Laplace expansion,

Inverse of a matrix by adjoint method, Cramer`s rule,

(iii) To study Gram–Schmidt orthogonalization process in an inner product space.

Number of lectures : 45

Unit I: Linear Transformations and Matrices (15 Lectures)

1. Review of linear transformations and matrix associated with a linear transformation: Kernel and image of a linear transformation, Rank– Nullity theorem (with proof), Linear isomorphisms and

its inverse. Any n–dimensional real vector space is isomorphic to RMatrix of sum, scalar multiple, composite and inverse of Linear transformations$_n$, Sum and scalar multiple of a linear.

transformation, spaceL(U,V) of Linear transformation from U to V where U and V are finite dimensional vector spaces over R , the dual space V*, linear functional, linear operator.

2. Elementary row operations, elementary matrices, elementary matrices are invertible and an invertible matrix is a product of elementary matrices.

3. Row space, column space of an $m\times n$ matrix, row rank and column rank of a matrix, Equivalence of the row and the column rank, Invariance of rank upon elementary row or column operations.

4. Equivalence of rank of an $m\times n$ matrix and rank of the linear transformation $L_A: \mathbb{R}^n \to \mathbb{R}^m$ $L_A(X) = AX$ where A is an $m\times n$ matrix. The dimension of solution space of the system of linear equation $AX = O$ equals $n-\text{rank}(A)$.

5. The solutions of non – homogeneous systems of linear equations represented by $AX = B$, Existence of a solution when $\text{rank}(A)= \text{rank}(A, B)$. The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.

Unit II: Determinants (15 Lectures)

1. Definition of determinant as an n–linear skew–symmetric function from $\mathbb{R}^n\times \mathbb{R}^n \times \ldots\times \mathbb{R}^n \to \mathbb{R}$ such that determinant of $(E_1, E_2,\ldots,E_n)$ is 1 where E_j denotes the j^{th} column of the identity matrix I_n. Determinant of a matrix as determinant of its column vectors (or row vectors).

2. of 2 Existence and uniqueness of determinant function via permutations, Computation of determinant $\times 2$, 3×3 matrices, diagonal matrices, Basic results on determinants such as $\det(A)=\det(A^t)$,

$\det(AB) = \det(A)\det(B)$. Laplace expansion of a determinant, Vandermonde determinant, determinant of upper triangular and lower triangular matrices, finding determinants by row reduction method.

3. Linear dependence and independence of vectors in $\mathbb{R}^n$ using determinants. The existence and uniqueness of the system $AX = B$ where A is an $n \times n$ matrix with $\det(A) \neq 0$. Cofactors and minors, Adjoint of an $n\times n$ matrix. Basic results such as $A(\text{adj } A) = \det(A) I_n$. An $n\times n$ real matrix is invertible if and only if $\det(A) \neq 0$. Inverse (if exists) of a square matrix by adjoint method, Cramer's rule.

4. Determinant as area and volume.

Unit III: Inner Product Spaces (15 Lectures)

1. Dot product in R^n, Definition of general inner product on a vector space over R. Examples of inner product including the inner product $<f, g> = \int_{-\pi}^{\pi} f(t)g(t)dt$ on $C[-\pi, \pi]$, the space of continuous real valued functions on $[-\pi, \pi]$.

2. Norm of a vector in an inner product space. Cauchy–Schwarz inequality, Triangle inequality, Orthogonality of vectors, Pythagoras's theorem and geometric applications in R^2, Projections on a line, The projection being the closest approximation, Orthogonal complements of a subspace, Orthogonal complements in R^2 and R^3. Orthogonal sets and orthonormal sets in an inner product space, Orthogonal and orthonormal bases, Gram–Schmidt orthogonalization process, Simple examples in R^2, R^3, R^4.

References:

1. Lang, Serge: Introduction to linear algebra. (2nd ed. 3rd Indian reprint) New Delhi. Springer (India) Private Limited, 2005(2009). 81-8128-260-6--(512.5Lan)

2. Kumaresan, S.: Linear algebra : a geometric approach. New Delhi. Prentice-Hall Of India Private Limited, 2001. 81-203-1628-2--(512.5KUM)

3. Krishnamurthy, V.; Mainra, V.P. & Arora, J.L: Introduction to linear algebra. New Delhi.Affiliated East-West Press Pvt. Ltd., 1976. 81-85095-15-9--(512.5KRI)

4. Lay, David C.: Linear algebra and its applications. (3rd ed.) Noida. Pearson Education Inc., 2016. 978-81-775-8333-5--(512.5Lay)

5. Artin, Michael: Algebra. (2nd ed. Indian Reprint) New Delhi. Prentice-Hall Of India Private Limited, 1994. 81-203-0871-9--(512ART)

6. Hoffman, Kenneth and Kunze, Ray: Linear algebra. (2nd ed.) Noida. Pearson India Education Services Pvt. Ltd, 2015. 978-93-325-5007-0--(512.5Hof)

7. Strang, Gilbert: Linear algebra and its applications. (3rd ed.) Fort Worth. Harcourt Brace Jovanovich College Publishers, 1988. 0-15-551005-3--(512.5STR)

8. Smith, Larry: Linear algebra. (3rd ed.) New York. Springer-Verlag, 1978.0-387-98455-0-- (512.5SMI)

9. Rao, Ramachandra A. and Bhimasankaran, P.: Linear Algebra. New Delhi. Tata Mcgraw- Hill Publishing Co. Ltd., 1992. 0-07-460476-7--(512.5RAO)

10. Banchoff, Thomas & Wermer, John: Linear algebra through geometry. (2nd ed.) New
York. Springer-Verlag, 1984. 0-387-97586-1--(512.5BAN/WER)

11. Axler, Sheldon: Linear algebra done right. (2nd ed.) New Delhi. Springer (India) Private Limited, 2010. 81-8489-532-2--(512.5Axl)

12. Janich, Klaus: Linear algebra.New Delhi. Springer (India) Private Limited, 1994. 978-81- 8128-187-6--(512.5Jan)

13. Bretscher, Otto: Linear algebra with applications.(3rd ed.) New Delhi. Dorling Kindersley (India) Pvt. Ltd, 2008. 81-317-1441-6--(512.5Bre)

14. Williams, Gareth: Linear algebra. New Delhi. Narosa Publishing House, 2009. 81-7319- 981-3--(512.9Wil)

Suggested Tutorials:

1. Rank–Nullity Theorem.
2. System of linear equations.

3. Calculating determinants of matrices, diagonal and upper / lower triangular matrices using definition, Laplace expansion and row reduction method, Linear dependence / independence of vectors by determinant concept.
4. Finding inverses of square matrices using adjoint method, Examples on Cramer's rule.
5. Examples of Inner product spaces, Orthogonal complements in R^2 , R^3, R^4. ^{2} and R^3 .
6. Examples on Gram–Schmidt process in R

****

S.Y.B.Sc. – Mathematics **Course Code: S.MAT.3.03**

Title: Discrete MATHEMATICS

Learning Objectives: To learn about (i) Advanced counting

(ii) Permutation and Recurrence Relation

(iii) Introductory Graph theory.

(iv) Introduction to probability measure

Number of lectures : 45

Unit I: Counting (15 Lectures)

1. Finite and infinite sets, Countable and uncountable sets, examples such as N, Z, $N\times N$, Q, $(0,1)$, R .
2. Addition and multiplication principle, Counting sets of pairs, two ways counting.
3. Stirling numbers of second kind, Simple recursion formulae satisfied by $S(n,k)$ and direct formulae for $S(n,k)$ for $k = 1,2,\cdots,n$.
4. Pigeon hole principle and its strong form, its applications to geometry, monotonic sequences.
5. Binomial and Multinomial Theorem, Pascal identity, examples of standard identities
6. Permutation and combination of sets and multi-sets, circular permutations, emphasis on solving problems.
7. Non-negative and positive integral solutions of equation $x_1 + x_2 + \cdots + x_k = n$.
8. Principle of Inclusion and Exclusion, its applications, derangements, explicit formula for d_n , various identities involving d_n , deriving formula for Euler's phi function $\varphi(n)$.
9. Permutation of objects, s_n composition of permutations, results such as every permutation is product of disjoint cycles; every cycle is product of transpositions, even and odd permutations, rank and signature of permutation, cardinality of S_n, A_n .

10. Recurrence relation, definition of homogeneous, non-homogeneous, linear and non-linear recurrence relation, obtaining recurrence relation in counting problems, solving (homogeneous as well as non-homogeneous) recurrence relation by using iterative method, solving a homogeneous recurrence relation of second degree using algebraic method proving the necessary result.

Unit II: Algorithms (15 Lectures)

1. Introduction to Algorithms and its Properties, Complexity of Algorithms.
2. Algorithms on Integers – Base b Expansions, Euclidean Algorithms, Horner's Algorithm, Finding primes, etc.
3. Algorithms on Matrices – Matrix Addition, Matrix Multiplication, Transpose of a Matrix, etc.
4. Recursive Algorithms – Fibonacci Series, Factorial of a non-negative number, etc
5. Searching Algorithms – Linear Search, Binary Search
6. Sorting Algorithms – Bubble Sort, Insertion Sort, Merge Sort
   (Use of Programming Language like PASCAL to aid in the implementation of the Algorithms)

Unit III: Graph Theory (15 Lectures)

1. Definition, examples and basic properties of graphs, pseudographs, complete graphs, bipartite graphs, Adjacency Matrix.
2. Isomorphism of graphs, paths and circuits, Eulerian circuits, Hamiltonian cycles, weighted graph.
3. Travelling salesman's problem, shortest path, Dijkstra's algorithm (Original and Improved).
4. Trees and their Properties, Spanning Trees, Kruskal's Algorithm, Prim's Algorithm.

Recommended Books:

1. Discrete Mathematics; Norman Biggs; Oxford University Press.
2. Introductory Combinatorics; Richard Brualdi; John Wiley and sons.
3. Discrete Mathematics with Graph Theory; Edgar Goodaire and Michael Parmenter; Pearson.
4. Discrete Mathematics and Its Applications; Kenneth H. Rosen; McGraw Hill Edition.
5. Introduction to Algorithms; Thomas H. Cormen, Charles E. Leiserson and Ronald L. Rivest; Prentice Hall of India

Additional Reference Books:

1. Combinatorics–Theory and Applications; V. Krishnamurthy; Affiliated East West Press.
2. Discrete Mathematics; Schaum's outline series.
3. Applied Combinatorics; Alan Tucker; John Wiley and Sons.
4. Discrete and Combinatorial Mathematics; Ralph P Grimaldi; Pearson international edition
5. Discrete Mathematics; Kenneth Ross and Charles Wright; Pearson.
6. Discrete Mathematical Structures; Bernard Kolman, Robert Busby, Sharon Ross; Prentice Hall India
7. Introduction to Graph Theory; Douglas B. West; Pearson
8. Concrete Mathematics-A Foundation for Computer Science; Ronald Graham, Donald Knuth and Oren Patashnik; Pearson Education.
9. How to Solve it by Computers, R.G. Dromey; Prentice-Hall India. 10. Graph Theory; Frank Harary; Narosa Publication

Suggested Practicals (3 practicals per week per batch):

1. Problems based on counting principles, Two way counting, Stirling numbers of second kind, Pigeon hole principle, Multinomial theorem, identities, permutation and combination of multiset.
2. Problems based on Derangement and rank signature of permutation, Recurrence relation, Inclusion-Exclusion principle. Euler phi function.
3. Algorithms on integers and prime numbers and on one dimensional arrays.
4. Algorithms on two dimensional arrays and matrices, While loop, G.C.D. etc.
5. Drawing a graph, checking if a degree sequence is graphical. Representing a given graph by an adjacency matrix and drawing a graph having given matrix as adjacency matrix.

6. Problems based on Eulerian and Hamiltonian graphs, Trees, Dijkstra's, Kruskal's and Prim's Algorithms
7. Miscellaneous theory questions from all units.

EVALUATION:-

CIA- I: 20 marks, 45 mins.
Unit I: Objectives/Short questions, not more than 5 marks each

CIA- II : 20 marks, 45 mins.
Unit II: Short questions/Presentation/Assignment, not more than 5 marks each

End Semester Exam – 60 marks, 2 hours OR Additional Exam – 100 marks, 3 hours

****