6th Semester Syllabus for Core and Applied Component Courses in Statistics St. Xavier's College, Mumbai. Revised June 2021



Syllabus For B.Sc. 6th Semester Courses in Statistics (June 2021 onwards)

Contents:

- Theory Syllabus for Courses:
 - o SSTA0601 Probability Distributions and Stochastic Processes
 - o SSTA0602 Statistical Inference.
 - o SSTA0603 Applied Statistics (IB)
 - o SSTA0604 Applied Statistics (IIB)
 - o SSTA06AC Optimization Methods in Operations Research
- Practical Course Syllabus for: SSTA06PR & SSTA06ACPR
- Evaluation and Assessment guidelines.

T.Y. B.Sc.

Course Code: SSTA0601

Title: Probability Distributions and Stochastic Processes.

Course Objectives:

- 1) To strengthen their concepts in mathematical statistics.
- 2) To prepare students to develop stochastic and queueing models.

Number of lectures: 60

Course Outcomes (COs):

- 1) Know bivariate moment generating function, and the definition and properties of the trinomial and multinomial distributions.
- 2) Grasp the bivariate normal distribution and its properties.
- 3) Use a procedure to test the significance of the population correlation coefficient.
- Know several stochastic processes, such as Poisson process, birth and death process, Yule' process, linear growth model and their applications.
- 5) View the queuing system as a stochastic process and obtain estimates for the various measures of effectiveness (performance measures) of the queuing model.

UNIT 1 BIVARIATE DISTRIBUTIONS:

(15 L)

i) Definition and properties of Moment Generating Function of two random variables of continuous and discrete type. Necessary and sufficient condition for independence of two random variables.

ii) Trinomial distribution:

Definition of joint probability distribution (X,Y). Joint moment generating function, moments μ_{rs} where r = 0,1,2 and s = 0,1,2

Marginal & conditional distributions. Their Means & Variances. Correlation coefficient between the random variables. Distribution of the Sum X+Y.

iii) Multinomial distribution:

Definition of joint probability distribution with parameters $(n, p_1, p_2, ..., p_{k-1})$ where $p_1 + p_2, +..., p_{k-1} + p_k = 1$. Other properties (Concept only)

UNIT 2 BIVARIATE NORMAL DISTRIBUTIONS:

(15 L)

Definition of joint probability distribution (X, Y). Joint moment generating function, moments μ_{rs} where r = 0,1,2 and s = 0,1,2Marginal & conditional distributions. Their Means & Variances. Correlation coefficient between the random variables. Condition for the independence of X and Y. Distribution of aX+bY, where a and b are constants.

SIGNIFICANCE OF CORRELATION COEFFICIENT:

Distribution of sample correlation coefficient when $\rho = 0$. Testing the significance of a correlation coefficient.

Fisher's z – transformation. tests for

i) $H_0: \rho = \rho_0$ and ii) $H_0: \rho_1 = \rho_2$. Confidence interval for ρ .

UNIT 3 <u>STOCHASTIC PROCESSES</u>:

Definition of stochastic process. Postulates and difference differential equations for the i) Poisson process ii) Pure birth process iii) Yule's process iv) Pure death process v) Poisson type of death process vi) Yule's type / Linear Markovian death process vii) Birth and death process viii) Linear growth model. Derivation of Pn(t), mean and variance wherever applicable.

UNIT 4 QUEUEING THEORY

(15L)

(15L)

Basic elements of the Queueing model. Roles of the Poisson and Exponential distributions. Derivation of Steady state probabilities for the birth and death process. Steady state probabilities and the various average characteristics for the following models

i) (M/M/1): $(GD/\infty/\infty)$ ii) (M/M/c): $(GD/\infty/\infty)$ v) (M/M/c): $(GD/N/\infty)$ tii) (M/M/c): $(GD/N/\infty)$ v) (M/M/c): (GD/N/N) c<N vi) $(M/M/\infty)$: $(GD/\infty/\infty)$ Derivation of the waiting time distribution for the (M/M/1): (FCFS/ ∞/∞) model

List Of Recommended Reference Books

- 1. Feller W: An introduction to probability theory and it's applications, Volume:1, Third edition, Wiley Eastern Limited.
- 2. Robert V. Hogg & Allen T. Craig: Introduction to Mathematical Statistics, Fifth edition, Pearson Education Pvt Ltd.
- 3. Alexander M Mood, Franklin A Graybill, Duane C. Boes: Introduction to the theory of statistics, Third edition, Mcgraw- Hill Series.
- 4. Hogg R. V. and Tanis E.A. Probability and Statistical Inference Fourth edition McMillan Publishing Company
- 5. S C Gupta & V K Kapoor: Fundamentals of Mathematical statistics, eleventh edition, Sultan Chand & Sons.
- 6. Taha H.A. Operations Research Mcmillan Publishing Co.
- 7. Kantiswaroop, P.K Gupta and Manmohan, Fourth edition, Sultan Chand & Sons.
- 8. Vohra N.D. Quantitative Techniques in Management Third edition McGraw Hill Companies
- 9. J Medhi: Stochastic Processes, Second edition, Wiley Eastern Ltd.
- 10. Biswas S. Topics in Statistical Methodology Wiley Eastern Ltd.
- 11. J. N. Kapur, H. C. Saxena Mathematical Statistics Fifteenth edition S. Chand and Company

Topics for Practicals

- 1. Trinomial & Multinomial distributions.
- 2. Bivariate Normal distribution.
- 3. Significance of correlation coefficient.
- 4. Stochastic processes.
- 5. Queueing theory

Evaluation (Theory): Total marks per course - 100.

CIA 40 marks CIA 1: Written test -20 marks CIA 2: Written test -20 marks End Semester Examination – 60 marks One question from each unit for 15 marks, with internal choice. Total marks per question with choice -20 to 25.

Evaluation of SSTA06PR (0601)

Total marks - 50. Group Project – 15 marks Journal – 5 marks. End Semester Practical Examination – 30 marks.

Grid Template - End Semester Examination (Theory)

Q. No	Knowledge (Definition, Descriptive Notes, Theoretical Proofs)	Understanding & Application (Illustration/Numerical Problems)	Marks
1.	15	05	20
2.	15	05	20
3.	15	05	20
Total	45	15	60
Weightage (%)	75%	25%	100%

T.Y. B.Sc. Title: Statistical Inference Course Code: SSTA0602

Course Objectives:

To empower students to validate assumptions made on population parameters.

Number of lectures: 60

Course Outcomes (COs):

- 1) Have an in-depth knowledge of the nuances of testing of a statistical hypothesis.
- 2) Comprehend the concept/ construction/ interpretation of the p-value, most powerful test, Neyman-Pearson fundamental lemma, uniformly most powerful test.
- 3) Know about 'likelihood ratio test' and the 'sequential probability ratio test'.
- 4) Understand the need for, types and uses of non-parametric tests.

Unit 1 TESTING OF HYPOTHESIS:

Statistical hypothesis. Problem of testing of hypothesis. Definitions and illustrations (15L) of i) Simple hypothesis ii) Composite hypothesis iii) Null Hypothesis iv) Alternative Hypothesis v) Test of hypothesis vi) Critical region vii) Type I and Type II errors viii) Level of significance ix) p-value. x) size of the test xi) Power of the test xii) Power function of a test. xiii) Power curve.

Definition of most powerful test of size α for a simple hypothesis against a simple alternative hypothesis. Neyman –Pearson fundamental lemma. Definition of uniformly most powerful (UMP) test. Construction of UMP test for one tailed alternative hypothesis. Randomized test.

Unit 2 <u>LIKELIHOOD RATIO TEST:</u>

Likelihood ratio principle. Definition of the test statistic and its asymptotic distribution (statement only) Derivation of the test procedure for testing a composite hypothesis against a composite alternative hypothesis for the parameters of Binomial, Poisson, Discrete & Continuous Uniform and Normal distribution.

Unit 3 NON-PARAMETRIC TESTS:

Need for non-parametric tests. Distinction between a parametric and a non parametric test. Concept of a distribution free statistic.

Confidence interval for a quantile.

One sample and Two sample non parametric test: (i) Sign test (ii) Wilcoxon Signed Rank test. (iii) Run test (iv) Kolmogrov Smirnov test.

(v) Median test (vi) Mann-Whitney-Wilcoxon test. (vii) Kruskal-Wallis test

(viii) Friedman test (ix) Fisher's Exact test

Assumptions, justification of the test procedure, critical regions for one tailed and two tailed test procedures. Problems with no ties.

(15L)

(15L)

Unit4 SEQUENTIAL PROBABILITY RATIO TEST:

Sequential probability ratio test procedures for testing a simple null hypothesis against a simple alternative hypothesis. Its comparison with fixed sample size. Most powerful test procedure. Definition of Wald's SPRT of strength (α , β). Problems based on standard distributions such as Bernoulli, Poisson, Normal, Exponential. Graphical and tabular procedures for carrying out the tests. O.C function and A.S.N function and their respective curves.

List Of Recommended Reference Books

- 1. Hogg R.V. and Craig A.T: Introduction to Mathematical Statistics Fourth edition London Macmillan Co. Ltd.
- 2. Hogg R.V. and Tanis E.A.: Probability and Statistical Inference. Third edition Delhi Pearson Education.
- 3. Daniel W.W.: Applied Non-Parametric Statistics First edition Boston-Houghton Mifflin Company
- 4. Sidney Siegal, N. John Castelian Jr. Nonparametric Statistics For Behavioral Sciences, Second edition McGraw Hill International editions
- 5. Wald A.: Sequential Analysis First edition New York John Wiley & Sons
- 6. Biswas S.: Topics in Statistical Methodology. First edition New Delhi Wiley eastern Ltd.
- 7. Gupta S.C. and Kapoor V.K.: Fundamentals of Mathematical Statistics Tenth edition New Delhi S. Chand & Company Ltd.
- 8. F.D. Gibbons: Non-Parametric Statistical Inference.

Topics for Practicals

- 1. Testing of Hypotheses.
- 2. Likelihood Ratio Tests.
- 3. Non-Parametric Tests.
- 4. Sequential Probability Ratio Test.

Evaluation (Theory): Total marks per course - 100.

CIA- 40 marks CIA 1: Written test -20 marks CIA 2: Written test -20 marks End Semester Examination – 60 marks One question from each unit for 15 marks, with internal choice. Total marks per question with choice -20 to 25.

Evaluation of SSTA06PR (0602)

Total marks - 50. Group Project – 15 marks Journal – 5 marks. End Semester Practical Examination – 30 marks (15L)

Grid Template - End Semester Examination (Theory)		
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Q. No	Knowledge (Definition, Descriptive Notes, Theoretical Proofs)	Understanding & Application (Illustration/Numerical Problems)	Marks
1.	15	05	20
2.	15	05	20
3.	15	05	20
Total	45	15	60
Weightage (%)	75%	25%	100%

T.Y. B.Sc. Title: Applied Statistics (I B) Course Code: SSTA0603

Course Objectives: To orient students on various applications of Statistics in industry

Number of lectures: 60

Course Outcomes (COs):

- 1) Understand the scope of Operations Research (OR) and its application to various industrial situations.
- 2) Know how to represent the problem of 'amount of inventory to be stocked' as a statistical problem based on the underlying assumptions and solve it to obtain optimal results.
- 3) Appreciate how the problem of 'when to replace an item' can be considered as a statistical problem which can be solved, under a given set of assumptions to achieve an optimal solution.
- 4) Know the techniques of decision making in the topic of Game Theory (decisions in situations involving competitive strategies).
- 5) Use the concept of simulation, and apply it to problems in the area of inventory, queuing and replacement.

 Unit 1
 INVENTORY CONTROL: Deterministic Models: Single item static EOQ models for i) Constant rate of demand with instantaneous replenishment, with and without shortages.
 15L

 ii) Constant rate of demand with uniform rate of replenishment, with and without shortages. iii) Constant rate of demand with instantaneous replenishment without shortages, with at most two price breaks.
 15L

 Probabilistic models. :

Single period with

- i) Instantaneous demand (discrete and continuous) without setup cost.
- ii) Uniform demand (discrete and continuous) without set up cost.

Unit 2 <u>REPLACEMENT THEORY:</u>

Replacement of items that deteriorate with time and the value of money:

- i) remains constant
- ii) changes with time (weighted average of costs method).

Replacement of items that fail completely.

Individual replacement and Group replacement policies.

Unit 3 GAME THEORY:

Definitions of Two-person Zero Sum Game, Saddle Point, Value of the Game, Pure and Mixed strategy. Optimal solution of two-person zero sum games: Dominance property, Derivation of formulae for (2×2) game. Graphical solution of $(2 \times n)$ and $(m \times 2)$ games. Reduction of Game Theory to LPP.

Unit 4 <u>SIMULATION:</u>

15L

15L

15L

Scope of simulation applications. Types of simulation. Monte Carlo Technique of Simulation. Elements of discrete event simulation. Generation of random numbers. Sampling from probability distribution. Inverse method. Generation of random observations from i) Uniform distribution ii) Exponential distribution iii) Gamma distribution. iv) Normal distribution. Simulation techniques applied to inventory and Queueing models.

List Of Recommended Reference Books

- 1. Sharma J. K.: Operations Research Theory and Application, Third edition, Macmillan India Ltd.
- 2. Sharma S.D.: Operations Research. Eleventh edition, Kedarnath, Ramnath & Co.
- 3. Kantiswaroop, P.K Gupta and Manmohan, Fourth edition, Sultan Chand & Sons.
- 4. V.K. Kapoor. Operations Research. -Techniques for Management. Seventh edition, Sultan Chand & Sons Educational Publishers New Delhi.
- 5. Taha H.A. Operations Research, Sixth edition, Prentice Hall of India Pvt Ltd.
- 6. Vohra N.D. Quantitative Techniques in Management Third edition McGraw Hill Companies
- 7. Bannerjee B. Operation Research Techniques, Second edition, Mumbai Business Books.
- 8. Bronson R. Operations Research, Shaum's Outline series
- 9. Smith P.J. Analysis of Failure and Survival Data.

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Topics for Practicals

- 1. Deterministic inventory models
- 2. Probabilistic inventory models.
- 3. Replacement Theory.
- 4. Game Theory.
- 5. Simulation

Evaluation (Theory): Total marks per course - 100.

CIA- 40 marks CIA 1: Written test -20 marks CIA 2: Written test -20 marks End Semester Examination – 60 marks One question from each unit for 15 marks, with internal choice. Total marks per question with choice -20 to 25.

Evaluation of SSTA06PR (0603)

Total marks - 50. Group Project – 15 marks Journal – 5 marks. End Semester Practical Examination – 30 marks

Grid Template - End Semester Examination (Theory)

Q. No	Knowledge (Definition, Descriptive Notes, Theoretical Proofs)	Understanding & Application (Illustration/Numerical Problems)	Marks
1.	15	05	20
2.	15	05	20
3.	15	05	20
Total	45	15	60
Weightage (%)	75%	25%	100%

T.Y. B.Sc. Title: Applied Statistics (II B)

Course Code: SSTA0604

Course Objectives:

To enable students to develop the technique of model building

Number of lectures: 60

Course Outcomes (COs):

- 1) Know of the fitting of a multiple linear regression (MLR) model with two independent variables.
- Understand the concept of autocorrelation, heteroscedasticity and multi-collinearity, the methods to detect the presence of these when fitting an MLR, and the consequences of using Ordinary Least Squares in its place.
- 3) Have clarity regarding the concept of a 'time series', and employ commonly used methods to estimate trend and seasonal component.
- 4) Understand the concept of reliability (survival function) and its use in various situations.

Unit 1 <u>MULTIPLE LINEAR REGRESSION – I :</u>

Concept of General Linear Model.

Introduction to Binary Logistic Regression.

Multiple linear regression model with two independent variables: Assumptions of the model, Derivation of ordinary least square (OLS) estimators of regression coefficients.

Properties of least square estimators (without proof)

Concept of multiple correlation, partial correlation, R² and adjusted R².

Properties of multiple and partial correlation coefficients.

Testing the significance of multiple and partial correlation coefficients.

Procedure of testing

- i) overall significance of the model
- ii) significance of individual coefficients
- iii) significance of contribution of additional independent variable to a model.

Confidence intervals for the regression coefficients

Residual analysis using graphs.

Unit 2 MULTIPLE LINEAR REGRESSION – II :

<u>Autocorrelation:</u> Concept, Detection using i) Run Test ii) Durbin Watson Test, Consequences of using OLS estimators in presence of autocorrelation, Generalized least square (GLS) method.

<u>Heteroscedasticity:</u> Concept,Detection using i) Spearman's rank correlation test ii) Breusch – Pagan – Godfrey Test. Consequences of using OLS estimators in presence of heteroscedasticity Weighted least square (WLS) estimators 15L

15L

<u>Multicollinearity</u>: Concept, Detection using R square & t ratios, simple correlation coefficients, Tolerance-Variance Inflation Factor (VIF) Consequences of using OLS estimators in presence of multi collinearity.

Unit 3 <u>TIME SERIES</u>:

15L

Definition of Time series. Its components. Models of Time Series. Estimation of trend by i) Freehand curve method ii) Method of semi averages iii) Method of moving averages iv) Method of least squares. v) Exponential smoothing method Estimation of seasonal component by i) Method of simple averages ii) Ratio to moving average method iii) Ratio to trend method

Unit 4 <u>RELIABILITY:</u>

15L

Concept of reliability or survival function, Hazard function, Cumulative hazard function

Life time distributions: i) Exponential ii) Gamma iii) Weibull iv) Gumbel. Definitions of increasing (decreasing) failure rate.

Observation schemes and censoring: left and right censoring, interval censoring, Type I, Type II, random right censoring. Kaplan-Meier estimator of survival function and median survival time.

Reliability: Structure function, coherent system, standard systems: series, parallel, k-out-of-n system of independent components having exponential life distributions. Mean Time to Failure of a system (MTTF).

List Of Recommended Reference Books

- 1) S.C.Gupta , V.K.Kapoor: Fundamentals of Applied Statistics, Third edition, Sultan Chand & Sons.
- 2) Barlow R.E. and Prochan Frank: Statistical Theory of Reliability and Life Testing, First edition, John Wiley & Sons.
- 3) Mann N.R., Schafer R.E., Singapurwalla N.D.: Methods for Statistical Analysis of Reliability and Life Data., First edition, Wiley International
- 4) Damodar Gujrathi: Basic Econometrics, Second edition McGraw-Hill Companies.
- 5) S.M.Ross: Probability Models & Applications.
- 6) A.M.Goon, M.K.Gupta, B.Dasgupta: Fundamentals of Statistics, Vol Two, Fifth Revised edition, The World Press Pvt Ltd.
- 7) Smith P.J: Analysis of Failure and Survival Data
- 8) Daniel W.W: Applied Non-Parametric Statistics First edition Boston-Houghton Mifflin Company

Topics for Practicals

- 1. Multiple regression model.
- 2. Autocorrelation, Heteroscedasticity, Multicollinearity.
- 3. Time series.
- 4. Reliability.

Evaluation (Theory): Total marks per course - 100.

CIA- 40 marks CIA 1: Written test -20 marks CIA 2: Written test -20 marks End Semester Examination – 60 marks One question from each unit for 15 marks, with internal choice. Total marks per question with choice -20 to 25.

Evaluation of SSTA06PR (0604)

Total marks - 50. Group Project – 15 marks Journal – 5 marks. End Semester Practical Examination – 30 marks

Grid Template - End Semester Examination (Theory)

Q. No	Knowledge (Definition, Descriptive Notes, Theoretical Proofs)	Understanding & Application (Illustration/Numerical Problems)	Marks
1.	15	05	20
2.	15	05	20
3.	15	05	20
Total	45	15	60
Weightage (%)	75%	25%	100%

T.Y.B.Sc. Statistics (Applied Component)

Title: Optimization Methods in Operations Research.

Course: SSTA06AC

THIS COURSE IS OFFERED TO STUDENTS WHO HAVE TAKEN STATISTICS UPTO THE 4 TH SEMESTER

Course Objectives:

To enable students to:

- 1. Obtain the optimal sequence of processing jobs
- 2. Understand the post optimality analysis in Linear Programming Problem (LPP)
- 3. Identify the nearest possible integer solution to a LPP
- 4. Identify solution to programming problems with certain pre-defined goals.
- 5. Obtain the optimal solution for some Non-Linear Programming Problems.

Number of lectures: 60 Lectures

Course Outcomes (COs):

At the end of course the students will be able to:

1. Plan and organise a set of jobs in the optimal manner.

2. Know the limits within which changes in the requirement coefficients and changes in the cost coefficients, will not affect the final optimal solution.

3. Understand the importance of an appropriate integer solution to a LPP.

4. Know how to solve problems with goals, apart from solving LPPs with requirement restrictions.

5. Solve Non-Linear Programming Problems.

Unit 1

SEQUENCING & SENSITIVITY ANALYSIS

Sequencing: Definition, Johnson's rule :Processing n jobs on two machines, Processing n jobs on three machines, Processing 2 jobs on k machines (graphical method). Maintenance of crew scheduling

Sensitivity analysis: Variation in the price vector 'c', Variation in requirement vector 'b'. Addition of a new variable to the LPP

Unit 2

INTEGER PROGRAMMING

Types of Integer Programming, Pure and Mixed Programming problems, zero-one model, Cutting–Plane Method, Branch–and–Bound Method. Traveling salesman problem.

Unit 3

GOAL PROGRAMMING

Definition, Goal programming with single goal, non-preemptive goal programming (both graphical and modified simplex method), preemptive goal programming (both graphical and modified simplex method).

Unit 4

AN INTRODUCTION TO NON - LINEAR PROGRAMMING

Formulation of non-linear programming, various types of non-linear programming problem, Kuhn Tucker conditions, the gradient procedure, quadratic programming.

List Of Recommended Reference Books:

- 1. Vora N.D, Quantitative Techniques in Management 3 rd Edition, (Tata McGraw Hill Co)
- 2. Taha H.A, Operations Research 8 th Edition (Macmillan Publishing Co)
- 3. Mokhtar S. Bazaraa, Hanif D. Sherali, C. M. Shetty, Non-Linear Programming Theory and Algorithms, 3 rd Edition, (Wiley Interscience)
- 4. Prem Kumar Gupta, Dr. D. S. Hira, Operation Research (S. Chand Publications)

(15 Lectures)

(15 Lectures)

(15 Lectures)

(15 Lectures)

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Topics for practicals:

- 1. Sequencing
- 2. Sensitivity Analysis
- 3. Integer Programming
- 4. Goal Programming
- 5. Non Linear Programming

Evaluation (Theory): Total marks per course - 100.

CIA- 40 marks CIA 1: Written test -20 marks CIA 2: Written test -20 marks End Semester Examination – 60 marks One question from each unit for 15 marks, with internal choice. Total marks per question with choice -20 to 25.

Evaluation of SSTA06AC

Practical: 50 Marks Practical Examination - 45 marks Journal: 05 marks

Grid Template - End Semester Examination (Theory)

Q. No	Knowledge (Definition, Descriptive Notes, Theoretical Proofs)	Understanding & Application (Illustration/Numerical Problems)	Marks
1.	15	05	20
2.	15	05	20
3.	15	05	20
Total	45	15	60
Weightage (%)	75%	25%	100%